AMERICAN UNIVERSITY OF BEIRUT

Profs. R. Chedid FACULTY OF ENGINEERING SUMMER TERM 2010-11
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Name:...................................... July 15, 2011

TEST ID: 1000

(EECE380) ENGINEERING ELECTROMAGNETICS

CLOSED BOOK (1 ½ HRS)

- Programmable Calculators are not allowed
- Provide your answers on the computer's card only
- Return the computer's card attached to the question sheet
- Mark with a pencil your last name, first name initial (FI) and father's name initial (MI).
- Mark your AUB ID NO. in the box titled "Social Security No."
- The test ID No. is your exam version. Mark it in the box titled ' Test ID".
- Use pencil for marking your answers
- When using eraser, be sure that you have erased well

Problem 1

Given $W=x^2y^2+xyz$. Compute the direction derivative dW/dl in the direction

$3a_x + 4a_y + 12a_z$ at (2,-1,0).

a. -35/17
b. 44/19
c. -44/13
d. -67/11
e. None of the above

Problem 2

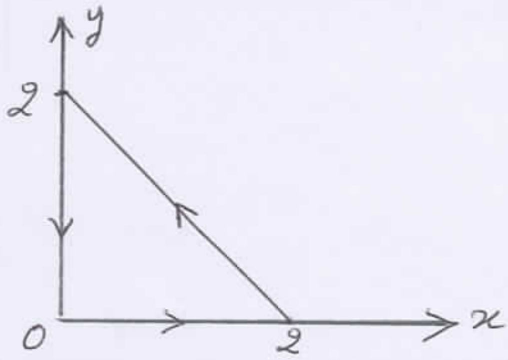
Given a vector field $F=y\, a_x + x\, a_y$, evaluate the integral $\oint F.dl$ from point P1(2,1,-1) to point P2(8,2,-1) along the straight line joining the 2 points.

a. 11
b. 12
c. 13
d. 14
e. None of the above

Problem 3

Assume a vector field $A=a_x\,(2x^2+y^2)+a_y\,(xy-y^2)$. Find $\oint(\nabla xA).d\mathbf{S}$ over the triangular area shown.

a. -4/3
b. -4/5
c. -4/7
d. -4/9
e. None of the above



Problem 4

Two point charges Q1 and Q2 are located at (0,5,-1) and (0,-2,6) respectively. Find the relation between Q1 and Q2 such that the total force on a test charge at the point P(0,2,3) will have no Y-component.

a. Q1/Q2= 3/4
b. Q1/Q2= 4/3
c. Q1/Q2= 5/3
d. Q1/Q2= 3/5
e. None of the above

Problem 5

A line charge of uniform density ρ_l forms a semicircle of radius b in the upper half xy-plane. Determine the magnitude and direction of the electric field intensity at the center of the semicircle.

a) $-a_y \rho_l / (4\pi \varepsilon_o b)$ V/m
b) $+a_y \rho_l / (2\pi \varepsilon_o b)$ V/m
c) $-a_y \rho_l / (2\pi \varepsilon_o b)$ V/m
d) $-a_y 2\rho_l / (\pi \varepsilon_o b)$ V/m
e) None of the above

Problem 6

Determine the work done in carrying a 5 μC charge from P1(1,2,-4) to P2 (-2, 8, -4) in the field

$E=a_x y+ a_y x$ along the parabola $y=2x^2$

a) 70 μJ
b) 80 μJ
c) 90 μJ
d) 100 μJ
e) None of the above

Problem 7

Region $y\leq 0$ consists of a perfect conductor while region $y\geq 0$ is a dielectric medium with $\varepsilon_{1r}=2$. If there is a surface charge of 2 nC/m^2 on the conductor. Determine D at (-4,1,5).

a) 4 a_y nC/m^2
b) 5 a_y nC/m^2
c) 1 a_y nC/m^2
d) 2 a_y nC/m^2
e) None of the above

Problem 8

A sphere of 200 mm radius contains electrical charge of density $2/(r \sin\theta)$ (C/m^3). What is the total charge contained within the sphere?

a) 0.789 C
b) 0.563 C
c) 0.391 C
d) 0.911 C
e) None of the above

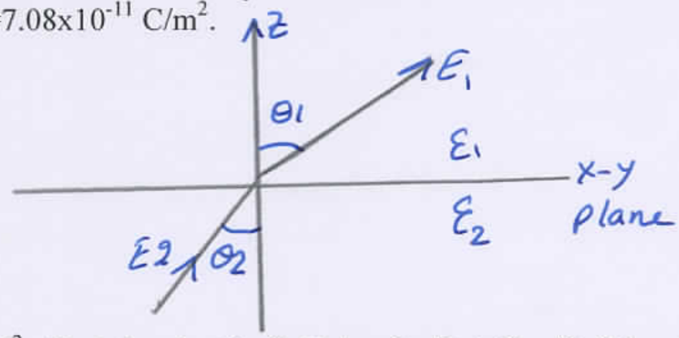
Problem 9

Given the potential function =2x+ 4y (V) in free space, find the stored energy in a $1 m^3$ volume centered at the origin.

a) $10^{-8}/(18\,\pi)$ J/m^3
b) $10^{-8}/(36\,\pi)$ J/m^3
c) $10^{-9}/(36\,\pi)$ J/m^3
d) $2x10^{-8}/(18\,\pi)$ J/m^3
e) None of the above

Problem 10

With reference to the figure below, find E1 if $E2= 3a_x - 2a_y + 4a_z$ V/m, $\varepsilon_1=2\varepsilon_0$, $\varepsilon_2=18\varepsilon_0$, and the boundary has a surface charge density $\rho_s=7.08x10^{-11}$ C/m^2.



a) $E1= 3a_x - 2a_y + 40a_z$ V/m
b) $E1= -3a_x - 2a_y + 4a_z$ V/m
c) $E1= -3a_x + 2a_y + 20a_z$ V/m
d) $E1= 3a_x + 2a_y + 40a_z$ V/m
e) None of the above

Problem 11

Given the potential $V=(10 \sin\theta \cos\phi)/R^2$. Find the electric field density **D** at (2, π/2, 0) in air.

a) 35.1 a_R pC/m^2
b) 22.1 a_R pC/m^2
c) 17.23 a_R pC/m^2
d) 11.6 a_R pC/m^2
e) None of the above

Problem 12

For a line charge $\rho_l=10^{-9}/2$ C/m on the z-axis, find V_{AB}, where A is (2m, π/2, 0) and B is (4m, π, 5m) in air.

a) 8.33V
b) 4.76 V
c) 6.24 V
d) 3.14 V
e) None of the above

EECE 380 : Engineering Electromagnetics

Solution of the Quiz of date July 15, 2011 - Summer term 2010-2011

Problem 1 :

$$\nabla W = (2xy^2 + yz)\vec{a}_x + (2x^2y + xz)\vec{a}_y + xy\,\vec{a}_z$$

$$\vec{a}_\ell = \frac{3}{\sqrt{3^2+4^2+12^2}}\vec{a}_x + \frac{4}{\sqrt{3^2+4^2+12^2}}\vec{a}_y + \frac{12}{\sqrt{3^2+4^2+12^2}}\vec{a}_z$$

$$= \frac{3}{13}\vec{a}_x + \frac{4}{13}\vec{a}_y + \frac{12}{13}\vec{a}_z$$

$$\nabla W \cdot \vec{a}_\ell = 4\left(\frac{3}{13}\right) + (-8)\left(\frac{4}{13}\right) + (-2)\left(\frac{12}{13}\right)$$

$$= \frac{-44}{13}$$

Problem 2 :

$$P_{12} = 6\,\vec{a}_x + 1\,\vec{a}_y$$

$$\vec{dl} = dx\,\vec{a}_x + dy\,\vec{a}_y$$

$$\int F.\vec{dl} = \int y\,dx + x\,dy = d(xy) = xy\Big|_1^2 = 16 - 2 = 14$$

$$\left.\begin{aligned} y &= \frac{1}{6}x + \frac{2}{3} \\ dy &= \frac{1}{6}dx \end{aligned}\right\} \Rightarrow \int\left(\frac{1}{6}x + \frac{2}{3} + x\,\frac{1}{6}\right)dx = \frac{1}{3}\frac{x^2}{2} + \frac{2}{3}x\Big|_2^8$$

$$= \frac{64-4}{6} + \frac{12}{3} = 14$$

Problem 3 :

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2+y^2 & xy-y^2 & 0 \end{vmatrix} = 0\,\vec{a}_x + 0\,\vec{a}_y + (y-2y)\,\vec{a}_z$$

$$\Rightarrow \nabla\times\vec{A} = -y\,\vec{a}_z$$

$\vec{ds} = dx\, dy$ $\qquad\qquad$ $y = -x + 2$

$$\iint \nabla \times \vec{A} \cdot \vec{ds} = \int_{x=0}^{2} \int_{y=0}^{y=-x+2} -y \, dx\, dy$$

$$= \int_0^2 -\frac{y^2}{2} \, dx$$

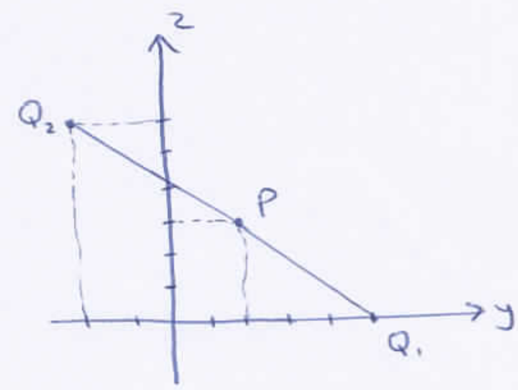
$$= \int_0^2 -\frac{1}{2}\left(x^2 + 4 - 2x \right) dx$$

$$= -\frac{1}{2}\left(\frac{x^3}{3} + 4x - 2x^2 \right)_0^2 = -\frac{1}{2}\left(\frac{8}{3} + 8 - 8 \right) = \frac{-4}{3}$$

Problem 4:

$$Q_2 \frac{4\vec{a}_y - 3\vec{a}_z}{5^3} + Q_1 \frac{-3\vec{a}_y + 4\vec{a}_z}{5^3}$$

$$4Q_2 - 3Q_1 = 0 \Rightarrow \frac{Q_1}{Q_2} = \frac{4}{3}$$

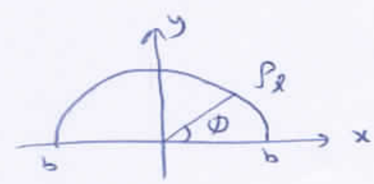


Problem 5:

$$dE = \frac{dQ}{4\pi \varepsilon R^2} \vec{a}_R$$

$$= \frac{\rho_\ell \, b \, d\phi}{4\pi \varepsilon \, b^2} \sin\phi \, (-\vec{a}_y)$$

$$= \frac{\rho_\ell}{4\pi \varepsilon b} \left(-\cos\phi \right)\Big|_0^\pi = \frac{\rho_\ell}{2\pi \varepsilon b} (-\vec{a}_y)$$



Problem 6:

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & 0 \end{vmatrix} = 0$$

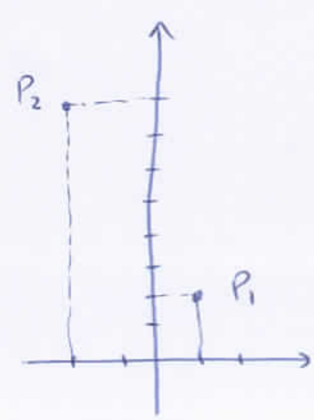
conservative

along $x=1$, y from 2 to 8 : $\vec{E} \cdot \vec{dl} = x \cdot D_y = 6$

along $y = 8$, x from 1 to -2 : $\vec{E} \cdot \vec{dl} = y \cdot Dx = -24$

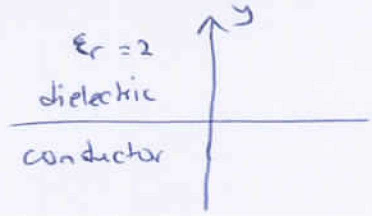
$\Rightarrow -\int \vec{E} \cdot \vec{dl} = 18\text{v}$

$QV = 5\mu C \times 18 = 90 \mu J$



Problem 7 :

$D = 2\,\vec{a_y} \quad nC/m^2$



Problem 8 :

$$\rho_v = \frac{2}{r \sin\theta} \quad C/m^3$$

$$Q = \iiint \rho_v \, dv = \iiint \frac{2}{r \sin\theta} \, r^2 \sin\theta \, dR \, d\theta \, d\phi$$

$$= \int 2r \, dr \, d\theta \, d\phi = r^2 (\pi)(2\pi)$$

$$= 2\pi^2 (.2)^2$$

$$= 0.04 \, (2\pi^2) = 0.8$$

Problem 9 :

$$V = 2x + 4y \Rightarrow -\nabla V = -\frac{\partial V}{\partial x}\vec{a_x} - \frac{\partial V}{\partial y}\vec{a_y}$$

$$\vec{E} = -2\vec{a_x} + 4\vec{a_y}$$

$$|E| = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{(\sqrt{20})^2}{36\pi\, 10^9} = \frac{10 \cdot 10^{-9}}{36\pi} = \frac{10^{-8}}{36\pi} \quad J/m^3$$

Problem 10 :

$$E_{n_2} = 4\vec{a_z}$$

$$E_{t_2} = 3\vec{a_x} - 2\vec{a_y} \quad , \quad E_{t_1} = E_{t_2}$$

$$\epsilon_1 E_{n_1} - \epsilon_2 E_{n_2} = \rho_s$$

$$2E_{n_1} = \frac{\rho_s}{\epsilon_0} + 18 E_{n_2} = \frac{7.08 \times 10^{-11}}{\frac{1}{36}10^{-9}} + 72 = 80$$

$$\Rightarrow E_{n_1} = 40$$

$$\Rightarrow E_1 = 3\vec{a_x} - 2\vec{a_y} + 40\vec{a_z} \quad V/m$$

Problem 11 :

$$V = \frac{10 \sin \theta \cos \phi}{R^2}$$

$$\vec{E} = -\nabla V = -\left\{ \frac{\partial V}{\partial R} \vec{a}_R + \frac{1}{R} \frac{\partial V}{\partial \theta} \vec{a}_\theta + \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi} \vec{a}_\phi \right\}$$

$$= -\left\{ -20 \sin \theta \cos \phi \, R^{-3} \vec{a}_R + \frac{10 \cos \theta \cos \phi}{R^3} \vec{a}_\theta + \frac{10 \sin \theta (-\sin \phi)}{R^3 \sin \theta} \vec{a}_\phi \right\}$$

$$\theta = \frac{\pi}{2} \quad , \quad \phi = 0$$

$$\Rightarrow \vec{E} = \frac{20}{R^3} \vec{a}_R + 0 + 0$$

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_r \frac{20}{R^3} \frac{10^{-9}}{36\pi} \vec{a}_R$$

$$= \varepsilon_r \frac{20}{8} \frac{10^{-9}}{36\pi} \vec{a}_R$$

$$= \varepsilon_r \, 0.0221 \times 10^{-9} \vec{a}_R = \varepsilon_r \, 22.1 \times 10^{-12} \vec{a}_R$$

$$= \varepsilon_r \, 22.1 \, \vec{a}_R \ \ pC/m^2$$

Problem 12 :

$$\frac{\rho_l}{2\pi\varepsilon} \ln \frac{r_2}{r_1} = \frac{10^{-9}}{2} \frac{1}{2\pi \varepsilon_r \frac{10^{-9}}{36\pi}} \ln 2$$

$$= \frac{9 \ln 2}{\varepsilon_r} = \frac{6.24}{\varepsilon_r}$$